

# Observation of $h_b(1P) \rightarrow \eta_b(1S)\gamma$

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## Abstract

We report the first observation of the radiative transition  $h_b(1P) \rightarrow \eta_b(1S)\gamma$ , where the  $h_b(1P)$  is produced in  $\Upsilon(5S) \rightarrow h_b(1P)\pi^+\pi^-$  dipion transitions. We measure the  $\eta_b(1S)$  mass to be  $(9401.0 \pm 1.9_{-2.4}^{+1.4}) \text{ MeV}/c^2$  with a width of  $(12.4_{-4.6}^{+5.5+11.5}) \text{ MeV}$  and a decay branching fraction of  $\mathcal{B}[h_b(1P) \rightarrow \eta_b(1S)\gamma] = (49.8 \pm 6.8_{-5.2}^{+10.9})\%$ . The measured  $\eta_b(1S)$  mass corresponds to a hyperfine splitting of  $(59.3 \pm 1.9_{-1.4}^{+2.4}) \text{ MeV}/c^2$ . This value deviates significantly from the current world average obtained from measurements of  $\Upsilon(3S) \rightarrow \eta_b(1S)\gamma$  and  $\Upsilon(2S) \rightarrow \eta_b(1S)\gamma$  reactions. We also report updated results for the  $h_b(1P)$  mass  $(9899.0 \pm 0.4 \pm 1.0) \text{ MeV}/c^2$  and its hyperfine splitting  $(0.8 \pm 1.1) \text{ MeV}/c^2$ . These measurements are performed using a  $121.4 \text{ fb}^{-1}$  data sample collected at the peak of the  $\Upsilon(5S)$  resonance with the Belle detector at the KEKB asymmetric-energy  $e^+e^-$  collider.

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## INTRODUCTION

Recently Belle reported the first observation of the  $h_b(1P)$  and  $h_b(2P)$  states [1]. The radiative transition to the  $\eta_b(1S)$  is expected to be one of the dominant decay modes of the  $h_b(1P)$ :  $ggg/\eta_b(1S)\gamma/gg\gamma = 57/41/2\%$  [2]. Belle's large  $h_b(1P)$  sample provide an opportunity to study the  $\eta_b(1S)$ , which is the ground state of the bottomonium system with  $b\bar{b}$  spin and orbital angular momentum equal to zero. The hyperfine splitting defined as  $\Delta M_{\text{HF}}[\eta_b(1S)] = M[\Upsilon(1S)] - M[\eta_b(1S)]$  provides a test of spin-spin interactions [3]. The  $\eta_b(1S)$  was first observed by BaBar [4, 5] and confirmed by CLEO [6]. Its mass is found to be higher than theoretical predictions [7, 8]. The tension between experimental results and predictions strongly motivates further experimental studies of the  $\eta_b(1S)$ . We note that no experimental information is available on the  $\eta_b(1S)$  width.

We report the first observation of the radiative transition  $h_b(1P) \rightarrow \eta_b(1S)\gamma$  and measurements of the  $\eta_b(1S)$  mass, width and decay branching fraction. We use a  $121.4\text{fb}^{-1}$  data sample collected at the peak of the  $\Upsilon(5S)$  resonance ( $\sqrt{s} \sim 10.865\text{ GeV}$ ) with the Belle detector [9] at the KEKB asymmetric-energy  $e^+e^-$  collider [10].

## METHOD

In the decay chain  $\Upsilon(5S) \rightarrow Z_b^+\pi^-$ ,  $Z_b^+ \rightarrow h_b(1P)\pi^+$ ,  $h_b(1P) \rightarrow \eta_b(1S)\gamma$  we reconstruct only the  $\pi^-$ ,  $\pi^+$  and  $\gamma$ . Here  $Z_b^+$  denotes the  $Z_b(10610)^+$  and  $Z_b(10650)^+$ , the two charged bottomonium-like resonances first identified by Belle in Ref. [11]. In this decay the typical momenta of the  $\pi^-$ ,  $\pi^+$  and  $\gamma$  are  $240\text{ MeV}/c$ ,  $730\text{ MeV}/c$  and  $500\text{ MeV}/c$ , respectively.

We define the missing mass of  $X$  (where  $X = \pi^+\pi^-$  or  $\pi^+\pi^-\gamma$ ) as

$$M_{\text{miss}}(X) = \sqrt{(E_{\text{c.m.}} - E_X^*)^2 - p_X^{*2}}, \quad (1)$$

where  $E_{\text{c.m.}}$  is the center-of-mass (c.m.) energy and  $E_X^*$  and  $p_X^*$  are the energy and momentum of the  $X$  system measured in the c.m. frame.

The  $\pi^+\pi^-\gamma$  combinations from the signal decay chain form a cluster in the  $M_{\text{miss}}(\pi^+\pi^-\gamma)$  versus  $M_{\text{miss}}(\pi^+\pi^-)$  plane centered at  $M[\eta_b(1S)]$  and  $M[h_b(1P)]$ , respectively (see Fig. 1). In this plane there is a vertical band due to correctly reconstructed  $\pi^+\pi^-$  combinations and misreconstructed  $\gamma$ 's, and a slanted band due to correctly reconstructed  $\gamma$ 's and misreconstructed  $\pi^+\pi^-$ . We introduce a new variable  $\Delta M_{\text{miss}}(\pi^+\pi^-\gamma) \equiv M_{\text{miss}}(\pi^+\pi^-\gamma) -$

$M_{\text{miss}}(\pi^+\pi^-) + m[h_b(1P)]$ . The advantage of the new variable is that the band with correctly reconstructed  $\gamma$ 's and misreconstructed  $\pi^+\pi^-$  combinations becomes horizontal (see Fig. 1) and the variables  $\Delta M_{\text{miss}}(\pi^+\pi^-\gamma)$  and  $M_{\text{miss}}(\pi^+\pi^-)$  are not correlated for signal events.

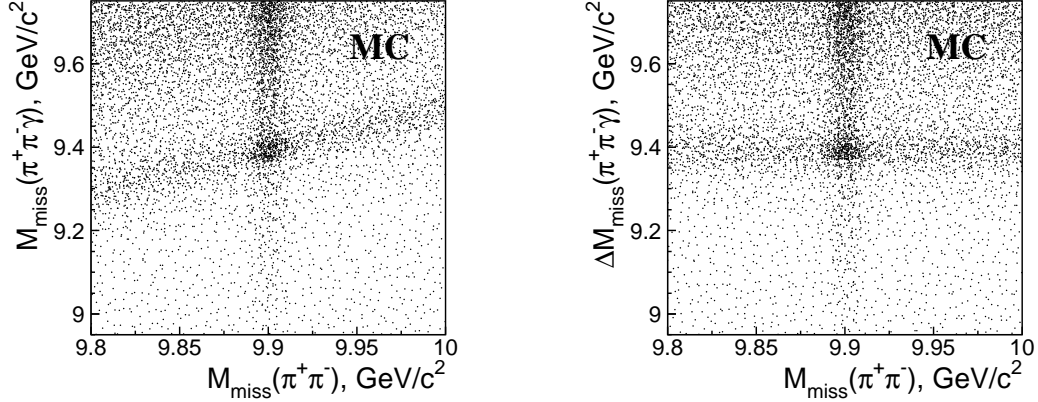


FIG. 1: Results of Monte-Carlo simulation for signal  $h_b(1P) \rightarrow \eta_b(1S)\gamma$  transitions.  $M_{\text{miss}}(\pi^+\pi^-\gamma)$  vs.  $M_{\text{miss}}(\pi^+\pi^-)$  distribution (left) and  $\Delta M_{\text{miss}}(\pi^+\pi^-\gamma)$  vs.  $M_{\text{miss}}(\pi^+\pi^-)$  distribution (right) for all  $\pi^+\pi^-\gamma$  combinations in the event.

It may be possible to perform a two dimensional fit to the  $\Delta M_{\text{miss}}(\pi^+\pi^-\gamma)$  vs.  $M_{\text{miss}}(\pi^+\pi^-)$  distribution. However, we follow a more intuitive approach. We divide the  $\Delta M_{\text{miss}}(\pi^+\pi^-\gamma)$  vs.  $M_{\text{miss}}(\pi^+\pi^-)$  plane into 10  $\text{MeV}/c^2$  wide horizontal slices, project each slice onto the  $M_{\text{miss}}(\pi^+\pi^-)$  axis and fit the  $M_{\text{miss}}(\pi^+\pi^-)$  distribution. We thus find the dependence of the  $h_b(1P)$  yield on the  $\Delta M_{\text{miss}}(\pi^+\pi^-\gamma)$  variable. We then search for the  $\eta_b(1S)$  signal as a peak in this distribution.

## SELECTION

We start with hadron event selection [12]. Continuum  $e^+e^- \rightarrow q\bar{q}$  ( $q = u, d, s, c$ ) background is suppressed by requiring that the ratio of the second to zeroth Fox-Wolfman moments satisfies  $R_2 < 0.3$  [13]. The  $\pi^+\pi^-$  selection requirements are the same as in the  $h_b(1P)$  and  $h_b(2P)$  observation paper [1]. We consider all positively identified  $\pi^+\pi^-$  pairs that originate from the vicinity of the interaction point. We require that the  $h_b(1P)$  is produced via an intermediate  $Z_b$ ,  $10.59 \text{ MeV}/c^2 < M_{\text{miss}}(\pi) < 10.67 \text{ MeV}/c^2$  [11]. This

requirement significantly reduces the background (by a factor of 5.2) without any significant loss of the signal. We consider all  $\gamma$  candidates, and apply a  $\pi^0$  veto,  $|M(\gamma\gamma_2) - m_{\pi^0}| > 13 \text{ MeV}/c^2$  with  $E_{\gamma_2} > 75 \text{ MeV}$ , where  $\gamma_2$  is any photon candidate in the event. The values of the cuts were optimized based on the figure of merit  $\text{FoM} = \frac{S}{\sqrt{B}}$ , where  $S$  is the number of signal events in the signal Monte-Carlo (MC),  $B$  is the number of background events estimated from a small fraction (0.1%) of data.

## STUDY OF INCLUSIVE $h_b(1P)$ SIGNAL

The fit to the inclusive  $M_{\text{miss}}(\pi^+\pi^-)$  spectrum (before combining  $\pi^+\pi^-$  and  $\gamma$  candidates) with the requirement of the intermediate  $Z_b$  is shown in Fig. 2. We use the same fit procedure

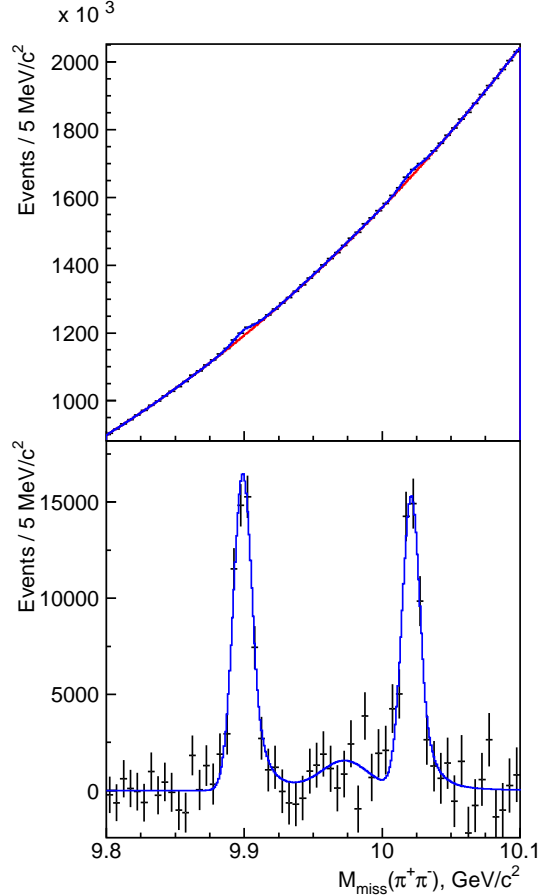


FIG. 2:  $M_{\text{miss}}(\pi^+\pi^-)$  spectrum with the requirement of an intermediate  $Z_b$  (top) and residuals (bottom).

as in Ref. [1]. The fit function consists of four components: the  $h_b(1P)$  signal, the  $\Upsilon(2S)$



signal, a reflection from the  $\Upsilon(3S) \rightarrow \Upsilon(1S)\pi^+\pi^-$  decay, and combinatorial background. The shapes of the peaking components are determined from the analysis of the  $\mu^+\mu^-\pi^+\pi^-$  data sample, that contains the  $\Upsilon(nS) \rightarrow \mu^+\mu^-$  ( $n = 1, 2, 3$ ) decays. The signals are found to have tails that account for about 8% of the yield and are due to the initial state radiation of soft photons. The  $h_b(1P)$  and  $\Upsilon(2S)$  intrinsic widths are negligible compared to the detector resolution, therefore the signals are described by a Crystal Ball function with width  $\sigma = 6.5 \text{ MeV}/c^2$  and  $6.8 \text{ MeV}/c^2$ , respectively. The width ( $\sigma$ ) of the  $h_b(1P)$  signal is determined from linear interpolation in mass from the widths of the  $\Upsilon(nS)$  peaks. The tail parameters of the  $h_b(1P)$  signal are assumed to be the same as for the  $\Upsilon(2S)$  signal. The  $\Upsilon(3S) \rightarrow \Upsilon(1S)\pi^+\pi^-$  reflection is described by a single Gaussian function with a width of  $\sigma = 18 \text{ MeV}$ . The combinatorial background is parameterized by a third order Chebyshev polynomial. We perform binned  $\chi^2$  fit using  $1 \text{ MeV}/c^2$  bins, though for clarity we display the data in  $5 \text{ MeV}/c^2$  bins. The results of the fit are given in Table I. The confidence level of this fit is 56%.

TABLE I: The yield and mass of the  $h_b(1P)$  from the fit to the inclusive  $M_{\text{miss}}(\pi^+\pi^-)$  spectrum.

	Yield, $10^3$	Mass, $\text{MeV}/c^2$
$h_b(1P)$	$(61.3 \pm 3.1^{+2.2}_{-0.3}) \times 10^3$	$(9899.0 \pm 0.4 \pm 1.0) \text{ MeV}/c^2$
$\Upsilon(3S) \rightarrow \Upsilon(1S)$	$(13 \pm 7) \times 10^3$	$9973.0 \text{ MeV}/c^2$
$\Upsilon(2S)$	$(54.8 \pm 3.9) \times 10^3$	$(10021.1 \pm 0.5) \text{ MeV}/c^2$

To estimate systematic uncertainty on the  $h_b(1P)$  parameters we vary the Chebyshev polynomial order (+1, +2); and fit range (we reduce it to  $9.98 \text{ MeV}/c^2$  and exclude all peaking components except for the  $h_b(1P)$  signal). We also introduce a correction factor for the signal width and allow it to float (we find  $f = 0.99 \pm 0.07$ ). We use a signal tail shape not only from the  $\Upsilon(2S)$  (the default case), but also from the  $\Upsilon(1S)$  and  $\Upsilon(3S)$ . A summary of the systematic uncertainties is given in Table II. For the mass measurement we introduce an additional  $\pm 1 \text{ MeV}/c^2$  uncertainty due to possible local variations of background shape as estimated in Ref. [1] using deviations of reference channels from the PDG values. The new value for the  $h_b(1P)$  mass corresponds to a hyperfine splitting  $\Delta M_{\text{HF}}[h_b(1P)] = (0.8 \pm 1.1) \text{ MeV}/c^2$ , where statistical and systematic uncertainties are added in quadrature.

TABLE II: Systematic uncertainties in the  $h_b(1P)$  parameters from various sources.

	Polynomial	Fit	Signal
	order	range	shape
$N[h_b(1P)], 10^3$	$+1.8$ $-0$	$+1.1$ $-0$	$+0.5$ $-0.3$
$M[h_b(1P)], \text{MeV}/c^2$	$+0.1$ $-0$	$+0.1$ $-0$	$+0$ $-0.1$

## EXTRACTION OF $\eta_b(1S)$ SIGNAL

To extract the  $\eta_b(1S)$  signal we fit the  $M_{\text{miss}}(\pi^+\pi^-)$  spectra in the  $\Delta M_{\text{miss}}(\pi^+\pi^-\gamma)$  bins. In the fit function we fix the masses of signals at the values given in Table I. We use a  $10 \text{ MeV}/c^2$  bin size in  $\Delta M_{\text{miss}}(\pi^+\pi^-\gamma)$ . The results for the  $h_b(1P)$ ,  $\Upsilon(2S)$  and  $\Upsilon(3S) \rightarrow \Upsilon(1S)$  reflection yields as a function of the  $\Delta M_{\text{miss}}(\pi^+\pi^-\gamma)$  are shown in Fig. 3. The  $h_b(1P)$  distribution shows a clear peak at  $9.4 \text{ GeV}/c^2$  that we identify as the  $\eta_b(1S)$  signal, while the other distributions do not exhibit significant structures.

We search for peaking backgrounds in the  $\Delta M_{\text{miss}}(\pi^+\pi^-\gamma)$  distribution of the  $h_b(1P)$  yield. We use a MC simulation for generic  $\Upsilon(5S)$  decays and consider separately  $h_b(1P) \rightarrow ggg$ ,  $h_b(1P) \rightarrow gg\gamma$  and  $e^+e^- \rightarrow \gamma_{\text{ISR}}\Upsilon(3S)$  processes. We do not find any sources of peaking background.

## CALIBRATION

We use the  $B^+ \rightarrow \chi_{c1}K^+$ ,  $\chi_{c1} \rightarrow J/\psi\gamma$ ,  $J/\psi \rightarrow e^+e^-/\mu^+\mu^-$  data sample for calibration. We require that the kaon and lepton candidates are positively identified and originate from the vicinity of the IP. For the  $J/\psi \rightarrow e^+e^-$  mode we attempt to reconstruct and recover bremsstrahlung photons. The mass window around the nominal  $J/\psi$  mass is  $\pm 30 \text{ MeV}/c^2$  ( $\pm 50 \text{ MeV}/c^2$ ) for the  $\mu^+\mu^-$  ( $e^+e^-$ ) mode. We perform a mass constrained fit to the  $J/\psi$  and  $\chi_{c1}$  candidates. We require  $|\Delta E| < 30 \text{ MeV}$  and  $M_{\text{bc}} > 5.27 \text{ MeV}/c^2$ , which are loose requirements. The  $\Delta E$  sidebands are defined as  $40 < |\Delta E| < 100 \text{ MeV}$ . The background is efficiently suppressed by the requirement  $\cos\theta_\gamma > -0.2$ , where  $\theta_\gamma$  is the helicity angle of the  $\chi_{c1}$  defined as the angle between the  $\gamma$  momentum and  $\chi_{c1}$  boost direction in the  $\chi_{c1}$  rest frame. The particular value of the cut is chosen so that the average  $\gamma$  energy in the c.m. frame is  $500 \text{ MeV}/c^2$ , i.e. is equal to the average energy of the photon in the  $h_b(1P) \rightarrow \eta_b(1S)$

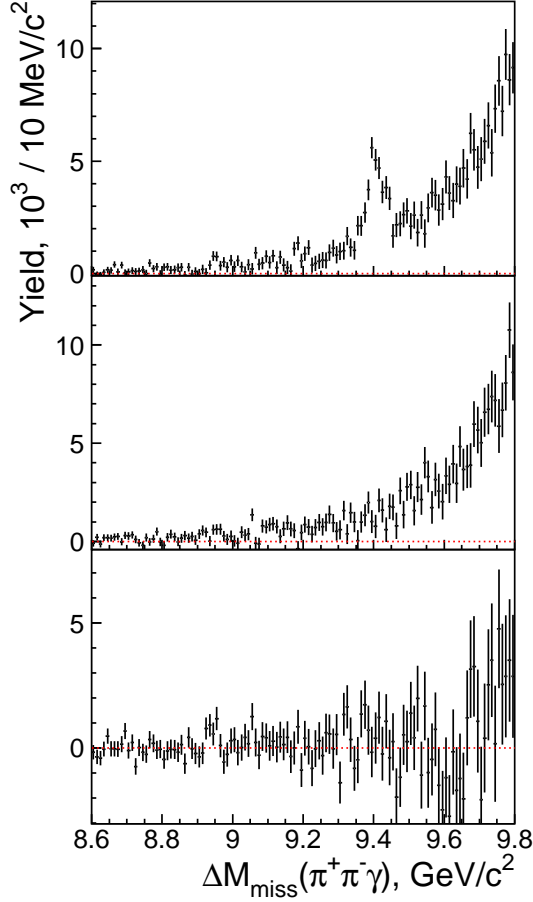


FIG. 3: The results for the  $h_b(1P)$  (top),  $\Upsilon(2S)$  (middle) and  $\Upsilon(3S) \rightarrow \Upsilon(1S)$  reflection (bottom) yields as a function of  $\Delta M_{\text{miss}}(\pi^+\pi^-\gamma)$ .

transition. The  $\Delta E$  distribution with the requirement  $3.44 < M(J/\psi\gamma) < 3.54 \text{ GeV}/c^2$  and  $M(J/\psi\gamma)$  distribution for the  $\Delta E$  signal region and for the normalized sidebands (see Fig. 4) indicate, that the background is small.

We parameterize the  $J/\psi\gamma$  mass distribution in MC by a Crystal Ball function [14] with an asymmetric core and with tails on both sides (8 parameters in total). This parameterization describes MC reasonably well (see Fig. 5). For the core we find  $\sigma_1 = 11.9 \text{ MeV}/c^2$  (left side) and  $\sigma_2 = 6.8 \text{ MeV}/c^2$  (right side).

We fit the  $M(J/\psi\gamma)$  distribution in data fixing the  $\sigma_1$  and  $\sigma_2$  parameters and introducing a shift of the peak position and width correction factor (see Fig. 5). We find for the shift:  $-0.7 \pm 0.3^{+0.2}_{-0.4} \text{ MeV}/c^2$  and for the width correction factor:  $1.15 \pm 0.06^{+0.05}_{-0.06}$ . The systematic uncertainty is estimated (1) by varying the fit interval; (2) by using only the left normalized

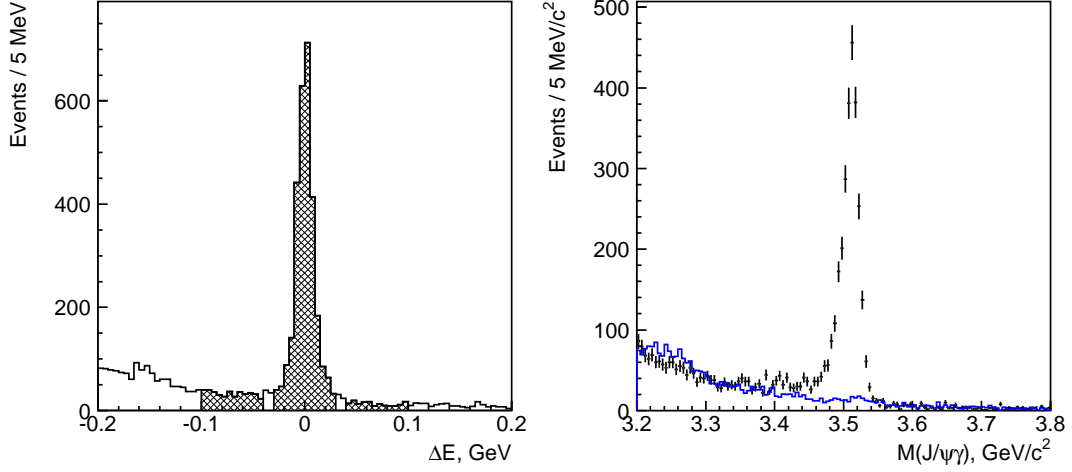


FIG. 4: (Left)  $\Delta E$  distribution for the selected  $B^+ \rightarrow \chi_{c1} K^+$  decay candidates; signal and sidebands regions are hatched. (Right)  $M(J/\psi\gamma)$  distribution for the  $\Delta E$  signal region (points with error bars) and for the normalized sidebands (open blue histogram).

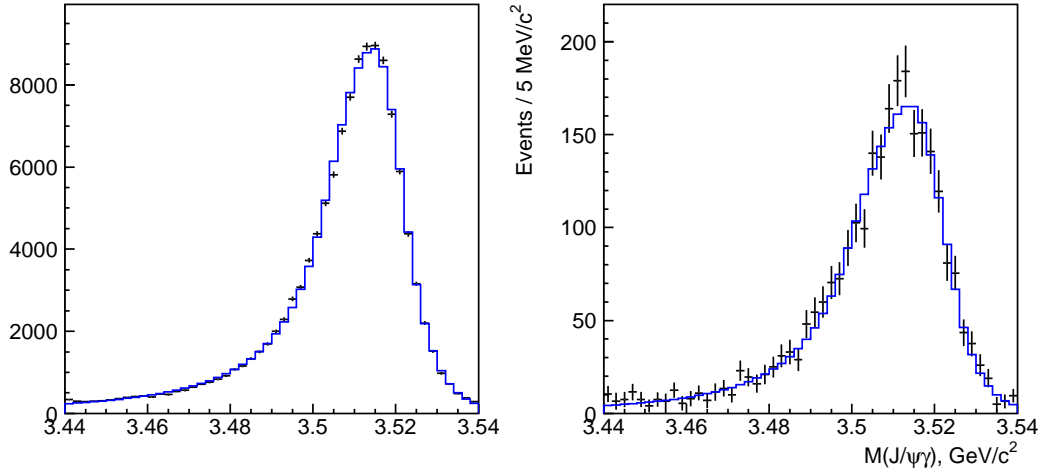


FIG. 5: The  $M(J/\psi\gamma)$  distribution in MC (left) and data  $\Delta E$  signal region with  $\Delta E$  sidebands subtracted (right). The fit is described in the text.

$\Delta E$  sideband for subtraction or only the right sideband; (3) by varying the  $\sigma_1/\sigma_2$  ratio in the parameterization: we use the  $\sigma_1/\sigma_2$  ratio from the fit to the distribution in (i) data and (ii) in the  $h_b(1P) \rightarrow \eta_b(1S)\gamma$  MC.

## $\eta_b(1S)$ MASS AND WIDTH

We fit the  $\Delta M_{\text{miss}}(\pi^+\pi^-\gamma)$  distribution to a sum of an  $\eta_b(1S)$  signal component and a combinatorial background contribution (see Fig. 6). The signal is a non-relativistic Breit-

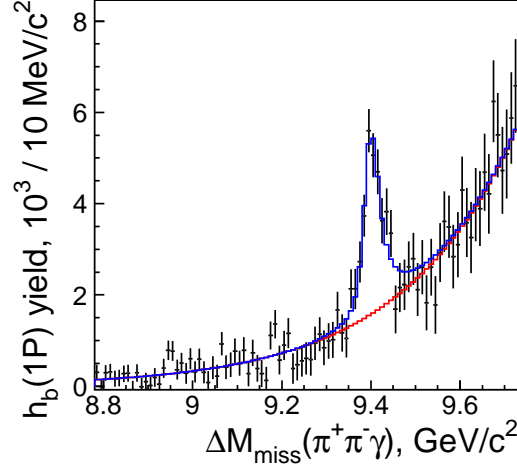


FIG. 6:  $\Delta M_{\text{miss}}(\pi^+\pi^-\gamma)$  distribution of the  $h_b(1P)$  yield with fit result superimposed.

Wigner (this parameterization is chosen to simplify comparison with BaBar and CLEO results) convolved with the calibrated resolution function (we use the shift and width correction factor determined from the  $\chi_{c1}$  signal in data). The combinatorial background is parameterized by an exponential function. We verify the fit procedure using MC. The fit results are shown in Table III. The confidence level of the fit is 77%. The significance of the

TABLE III: The yield, mass and width of the  $\eta_b(1S)$  signal from the fit to the  $\Delta M_{\text{miss}}(\pi^+\pi^-\gamma)$  distribution of the  $h_b(1P)$  yield.

$N[\eta_b(1S)]$	$(21.9 \pm 2.0^{+5.6}_{-1.7}) \times 10^3$
$M[\eta_b(1S)]$	$(9401.0 \pm 1.9^{+1.4}_{-2.4}) \text{ MeV}/c^2$
$\Gamma[\eta_b(1S)]$	$(12.4^{+5.5+11.5}_{-4.6-3.4}) \text{ MeV}$

$\eta_b(1S)$  signal is  $14\sigma$ .

To estimate the systematic uncertainty we vary the fit range in the fits to the  $M_{\text{miss}}(\pi^+\pi^-)$  distributions (instead of the default range  $9.8 - 10.1 \text{ GeV}/c^2$  we use  $9.8 - 9.98 \text{ GeV}/c^2$ ); we also vary the polynomial order in these fits (we increase the order by one and by two); we

vary the binning of the  $\Delta M_{\text{miss}}(\pi^+\pi^-\gamma)$  distribution (we scan the starting point of bin with 1 MeV/ $c^2$  steps); we also vary the fit range in the fits to the  $\Delta M_{\text{miss}}(\pi^+\pi^-\gamma)$  distribution; we vary the parameterization of the combinatorial background in the fits to the  $\Delta M_{\text{miss}}(\pi^+\pi^-\gamma)$  distribution (in addition to a single exponential we use the sum of two exponentials, second and third order polynomials, we also use the function  $\exp(\sum_{k=0}^n c_k x^k)$ , i.e. an exponential of a polynomial, with  $n = 2, 3, 4$ ); to take into account the uncertainty in the resolution function we vary the width correction factor by  $\pm 1 \sigma$  (we combine its statistical and systematic uncertainty in quadrature) and we take into account the uncertainty in the mass shift parameter; we vary the selection criteria [ $R_2$  cuts: 0.25, 0.3 (default), 0.35, 0.4;  $\pi^0$  veto, cut on  $|M(\gamma\gamma_2) - M_{\pi^0}|$ : 10, 13 (default), 15 MeV/ $c^2$ , cut on  $E_{\gamma 2}$ : 50, 75 (default), 100 MeV]; we take into account the uncertainty in the  $h_b(1P)$  mass. A summary of the systematic uncertainties is given in Table IV. To obtain the total systematic uncertainty we add all

TABLE IV: Systematic uncertainties in the  $\eta_b(1S)$  parameters from various sources.

	$N[\eta_b(1S)], 10^3$	$M[\eta_b(1S)], \text{MeV}/c^2$	$\Gamma[\eta_b(1S)], \text{MeV}$
Range in $M_{\text{miss}}(\pi^+\pi^-)$ fits	+0.0 -0.6	+0.0 -0.2	+0.0 -0.1
Poly order in $M_{\text{miss}}(\pi^+\pi^-)$ fits	+0.0 -0.6	+0.1 -0.1	+0.0 -0.4
$\Delta M_{\text{miss}}(\pi^+\pi^-\gamma)$ binning	+0.2 -0.1	+0.3 -0.8	+1.0 -0.8
Range in $\Delta M_{\text{miss}}(\pi^+\pi^-\gamma)$ fits	+0.9 -0.2	+0.1 -0.1	+1.4 -0.3
$\Delta M_{\text{miss}}(\pi^+\pi^-\gamma)$ bg parameterization	+5.5 -1.4	+0.5 -0.2	+10.9 -2.2
Resolution function	+0.2 -0.1	+0.5 -0.5	+1.4 -1.4
Selection requirements	—	+0.4 -1.9	+3.0 -2.0
$h_b(1P)$ mass	—	+1.1 -1.1	—
Total	+5.6 -1.7	+1.4 -2.4	+11.5 -3.4

sources in quadrature.

We study the shift of the  $\eta_b(1S)$  parameters in case other line-shape parameterizations are used. We consider the KEDR parameterization [15]:  $BW(m) \frac{E^3 E_0^2}{E E_0 + (E - E_0)^2}$ , where  $BW(m)$  is the Breit-Wigner function,  $E$  [ $E_0$ ] is the  $\gamma$  energy in the  $h_b(1P)$  rest frame [calculated for the pole mass of the  $\eta_b(1S)$ ]. We also consider the CLEO parameterization [16]:  $BW(m) E^3 \exp(-\frac{E^2}{8\beta^2})$ , where  $\beta$  is a fit parameter. Both the KEDR and CLEO Collaborations used these parameterizations for the  $J/\psi \rightarrow \eta_c \gamma$  transitions. We do not find

considerable shifts in the  $\eta_b(1S)$  yield, mass or width if these alternative parameterizations are used instead of the non-relativistic Breit-Wigner function.

The significance of the  $\eta_b(1S)$  signal including systematic uncertainties is  $13\sigma$ .

## MEASUREMENT OF $\mathcal{B}[h_b(1P) \rightarrow \eta_b(1S)\gamma]$

We measure the  $\eta_b(1S)$  [ $h_b(1P)$ ] yield in the events that fail the  $R_2 < 0.3$  or  $\pi^0$  veto requirements [ $R_2 < 0.3$  requirement]. In the fit to the  $\Delta M_{\text{miss}}(\pi^+\pi^-\gamma)$  [ $M_{\text{miss}}(\pi^+\pi^-)$ ] distribution for the rejected events we allow the  $\eta_b(1S)$  mass and width [the  $h_b(1P)$  mass] to float within the uncertainties of the measured values given in Table III (Table I). We find  $N_1[\eta_b(1S)] = (5.5 \pm 2.7 \pm 2.1) \times 10^3$  and  $N_1[h_b(1P)] = (13.0 \pm 2.8 \pm 0.5) \times 10^3$ .

From the  $N_1$  values and the yields from Tables I and III we determine the total yields  $N_0[h_b(1P)]$  and  $N_0[\eta_b(1S)]$  without requirements on  $R_2$  or a  $\pi^0$  veto. We obtain  $\mathcal{B} = N_0[\eta_b(1S)]/N_0[h_b(1P)]/\epsilon$ , where  $\epsilon$  is the reconstruction efficiency of the radiative photon, which is found from MC to be 74.1% with 2% systematic uncertainty (the MC statistical uncertainty is negligible). We find

$$\mathcal{B}[h_b(1P) \rightarrow \eta_b(1S)\gamma] = (49.8 \pm 6.8^{+10.9}_{-5.2})\%.$$

## CONCLUSIONS

We report the first observation of the radiative transition  $h_b(1P) \rightarrow \eta_b(1S)\gamma$ , where the  $h_b(1P)$  is produced in  $\Upsilon(5S) \rightarrow h_b(1P)\pi^+\pi^-$  dipion transitions. We report the single most precise measurement of the  $\eta_b(1S)$  mass,  $(9401.0 \pm 1.9^{+1.4}_{-2.4}) \text{ MeV}/c^2$ , which corresponds to the hyperfine splitting  $\Delta M_{\text{HF}}[\eta_b(1S)] = (59.3 \pm 1.9^{+2.4}_{-1.4}) \text{ MeV}/c^2$ . This value deviates significantly from the current world average [17] but decreases tension with theoretical expectations [7, 8] (see Fig. 7). We report the first measurement of the  $\eta_b(1S)$  width  $(12.4^{+5.5+11.5}_{-4.6-3.4}) \text{ MeV}$ , which is in the middle of the range of predictions from potential models, 4 – 20 MeV [18]. For the branching fraction we find  $\mathcal{B}[h_b(1P) \rightarrow \eta_b(1S)\gamma] = (49.8 \pm 6.8^{+10.9}_{-5.2})\%$  in agreement with expectations [2].

We also report updated results for the  $h_b(1P)$  mass  $(9899.0 \pm 0.4 \pm 1.0) \text{ MeV}/c^2$  and hyper-fine splitting  $\Delta M_{\text{HF}}[h_b(1P)] = (0.8 \pm 1.1) \text{ MeV}/c^2$ . The latter is consistent with zero, as expected.

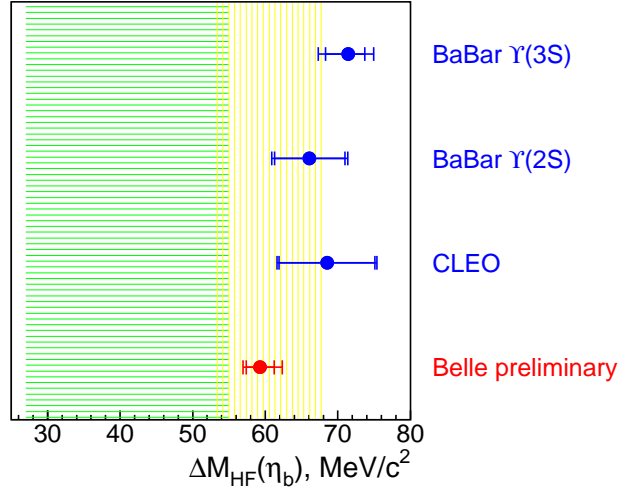


FIG. 7: Hyperfine splitting measured by BaBar in  $\Upsilon(3S)$  data [4], BaBar  $\Upsilon(2S)$  data [5], CLEO [6] and present preliminary result of Belle. In addition, pQCD [7] (horizontally hatched) and Lattice QCD [8] (vertically hatched) predictions are shown.

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